MATHEMATICAL DISCOVERY IN THE CLASSROOM –

FUNDED BY THE CHICAGO SHAY GRANT

THEMES

- **Do**: Explore, Experiment, Investigate, Problem Solve, Collaborate, Allow Open-ended Flexible Learning, Encourage Discovery, Emphasize History and Context.
- **Don't:** Memorize, Imitate, Focus on Terminology, Turn Math into Recipes, Emphasize Syntax over Semantics.

ICJA Workshop

ONLINE LECTURES

My name is Ali Qureshi. I'm a student of Computer Science of FAST NU University from Pakistan. A time ago I hated computer science as my field which gradually went the opposite and I love it today but in particular, I was hating Theory Of Computation/Automata so much that when my teacher used to enter the class I wanted throw a water balloon on him. But thanks to your conceptual building lectures on the subject I will research in this field in future.

I wrote this because as my country is counted as one of the third world countries having not that much educated people and if there are educated people they've not that teaching skill that is required to be a Lecturer. And with not that much educational resources we find people like you on the website sharing your knowledge and information which helps people like me to build interest in Computer Science.

I wrote this to thank you for making this subject beautiful and amazing for me and maybe many people from my country where sadly Youtube is blocked are getting help from your lectures.

EXAMPLE 1

Babylonian Method For Calculating Square Roots

Square Root of 200. Make any reasonable guess 10.

Try it. 200/10 = 20.

20 is too high, 10 is too low so try their average: 15.

Rinse, lather, repeat.

Try 15. 200/15 = 13 1/3, so 15 is too high, but 13 1/3 is low

Try their average: 14 1/6 = 14.16666

200/14.16666 = 14.1176. Now we are getting close.

EXAMPLE 1

Investigate

- Why does this work?
- How many steps to get the answer correct to two decimal places? What does this depend on?
- Can we modify the idea for cube roots? How?
- Compare this to other methods you can find.

EXAMPLE 2

For square roots, the Babylonians effectively used the feedback equation:

$$New = (N/Old + N) /2$$

Consider this strange looking feedback equation:

$$New = 1/(3 - Old)$$

Experiment: Old = 1 gives the sequence:

1/2 2/5 5/13 13/34 34/89 etc.

EXAMPLE 2

Investigate

1/2 2/5 5/13 13/34 34/89 …

Does this sequence decrease? (Yes.)

Does it converge to some value? (Yes. $(3-\sqrt{5})/2$)

What connection is their between this sequence and the famous Fibonacci numbers: 1 1 2 3 5 8 13 21 34 55 89 etc.?

Motivates quadratic equations and solutions, continued fractions, and convergent sequences.

6TH GRADE EXAMPLE

Phone Number Trick

- Multiply the first three digits by 80; Add one
- Multiply by 250
- Add the last 4 digits.
- Repeat step 4,
- Subtract 250; Divide by 2

$[(80x + 1)*250 + 2y – 250]/2 = 20000x+2y = 10000x +y$

6TH GRADE EXAMPLE

A Candy Game

A number of students sit in a circle, each with an even number of pieces of candy. Each time the teacher blows a whistle, each student simultaneously gives half of his or her own candy to the neighbor on the right. Any student who ends up with an odd number of pieces of candy gets one more piece from the teacher.

Explore: Everyone ends up eventually with the same number of pieces of candy. Why? How does this number depend on the number of people in the circle, and/or the pieces each starts with?

9TH GRADE EXAMPLE

Take a deck of cards with any number of cards between 16 and 20.

Choose two numbers A and B, each between 10 and 15

"Shuffle" A, B, A

Experiment: What happens? Why?

VISUALIZATION

For example A B C D E F G H I J K L M N O

- Choose 13 and 11
- After 13: N O M L K J I H G F E D C B A
- After 11: D C B A E F G H I J K L M O N
- After 13: O N M L K J I H G F E A B C D

Investigate: Do the numbers have to be between 10 and 15 to force the last card to the top? What are the actual required conditions?

9TH GRADE EXAMPLE

Can we make a card trick out of this?

Split a deck of cards into three equal parts. Each person chooses a card and places it on the bottom of the deck.

Each person chooses a secret number, shuffles, and passes left. Each person shuffles new deck, and passes right. Each person shuffles once more. Where are the original chosen cards?

LOTS OF EXAMPLES

Weekly Puzzle at

Rediscovering Mathematics' Facebook Page